An Account *Concerning the* Reſolution *of* Equations in Numbers; *imparted by Mr.* Iohn Collins.

This Account ſhould have been annex'd to what was diſcourſed of Monſieur *Sluſius* his *Meſolabe* in the precedent Tract, if then we had found room for it. For, the Reader having there underſtood, how farr the *Geometrick* part of *Algebra* is advanc'd by that excellent perſon, 'twas likely, he would be inquiſitive to hear ſomewhat concerning the *Exegeſis Numeroſa*, or the Reſolution of Æquations in *Numbers*. For whoſe ſatisfaction herein, we ſhall here inſert the Account then omitted, being part of a narrative, formerly made by M. *Iohn Collins* touching ſome late Improvements of Algebra in *England*, upon the occaſion of its being alledged, that none at all were made ſince *Des Cartes*.

1. It hath been obſerv'd by divers of this Nation, that in any Æquation, howſoever affected, if you give a Root, and find the Abſolute number or Reſolvend (which *Vieta* calls *Homogeneum Comparationis*) and again give more Roots and find more Reſolvends, that if theſe Roots or rather rank of Roots be aſſum'd in *Arithmetical* progreſſion, the Reſolvends, as to their firſt, ſecond or third differences, *&c.* imitate the Laws of the pure Powers of an Arithmetical progreſſion of the ſame degree, that the higeſt Power or firſt term of the Equation is of. e. g. In this Equation $aaa - 3aa + 4a = N$,

				1.*dif*	2.*dif*	3.*dif.*
If *a* be =	10	Then N. or the Abſolutes or Reſolvends will be found to be	740			
	9		522	218		
	8		352	170	48	
	7		224	128	42	6
	6		132	92	36	6

To wit the 3d. differences of thoſe *Abſolutes* are equal, as, in the Cubes of an Arithmetical Progreſſion.

2. To find, what habitude thoſe *differences* have to the *Coefficient*, of the Equation, 'ist beſt to begin from an Unit.

3. In any Arithmetical Progreſſion, if you multiply Num-

bers by *pairs*, you ſhall create a rank of Numbers whoſe 2d. differences are equal; and if by *ternaries*, then the 3d. differences of thoſe Products ſhall be equal. And how to find the greateſt Product of an Arithmetical Progreſſion of any number of terms having any common difference aſſign'd, contain'd in any Number propoſed, is ſhew'd by *Paſcal* in his Tract *du Triangle Arithmetique*, where he apply's it to the Extraction of the Roots of ſimple powers.

4. It appears, How this rank may be caried eaſily by Addition, till you have a *Reſolvend* either equall or greater or leſſe, than that propoſed.

5. When you have a *Majus* and *Minus*, you may interpoſe as many more termes in the Arithmetical Progreſſion as you will, that is to ſay, Subdivide the Common difference in the Arithmetical Progreſſion, and render it leſſe; and then renew, and find the Reſolvends, which are eaſily obtain'd out of the Powers and their Coefficients, which are ſuppoſ'd knowne, and may be readily raiſ'd from a *Table* of *Squares* and *Cubes*, &c. with which kind the *Reader* may be furniſht in *Guldini Centrobaryca* and *Babingtons Fireworks*: By this means you may obtain divers Figures of the Root; and then the General Method of *Vieta* and *Harriot* runs away more eaſily, and is ſo far improv'd, that after any figure is plac'd in the Root, moſt certain Characters are given to know by aide of the ſubſequent *Dividend* and *Diviſor*, Whether the figure before aſſum'd be too great or too ſmall: or laſtly it may well be concluded, that, as in Logarithmes, when you propoſe ſuch an one as is not abſolutely given in the *Canon*, you doe by Proportional Work, uſing the aid of their firſt differences (when their Abſolute Numbers differ by *Unit*) find the abſolute Number true to 5. or 6. places further than the *Canon* gives it (the reaſon whereof is, that the firſt Differences doe likewiſe agree to about the ſame Number of places;) that I ſay, the like may be done in Æquations, after divers of the firſt figures of the root are found; provided there be the like agreement in the firſt differences of the interpoled Reſolvends.

Moreover we ought here to take notice of a more ſubtile kind of Interpolation, common to all gradual Ranks or Progreſſions of Numbers, wherein Differences happen to be equal: Of which

kind

kind the *Reader* may find Examples in *Briggii Arithmetica Logarithmica et Trigonometria Britannica*, relating to Logarithmes, Sines, and the Powers of an Arithmetical Progreſſion: But the method there deliver'd may be rendred more eaſy and general, *viz.* by aid of a *Table* of figurat Numbers, by deriving Generating differences *ſought*, from thoſe *given*; a doctrine, that eaſily flows from *Mercators Logarithmotechnia*, and of uſe in the Caſe in hand, ſhould we ſuppoſe theſe Powers and their Coefficients vnknown, or a *Table* of *Squares* and *Cubes* wanting, and give nothing more, than a few *Reſolvends* belonging to equal Moments or Spaces. And this may likewiſe be of good uſe in *Guaging*, when having the Contents of a Solid, for every 3. Inches, more or leſſe, given, without knowing the dimenſions of the Figure, and even in moſt Caſes, when the differences are Progreſſive of one kind, without knowing the Figure it ſelf, having nothing given but its Contents at ſeveral equal Parallel diſtances, each ſuch diſtance may be ſubdivided, and made as many as you pleaſe, and the reſpective Contents found by this general Method of Interpolation.*

After one Root is obtained, the Methods of *Huddenius* and others will depreſſe the Equations ſo as to obtain more, and conſequently all of them.

6. It is eaſy, by a *Table* of figurate Numbers to give the ſum of any ſuch Rank or any term in it relating to a known part of the *Series* of Equals or Roots; but *è converſo*, giving the *Reſolvend* to find the Root, coms to an Equation as difficult as that propoſed; as in D. *Wallis* his Chapter of Figurate Numbers.

7. Some affirm, they can give good Approaches for the obtaining a Root of any *pure* power, *affected* Equation, or for the finding of any of the mean Proportionals in any Rank between two extreams given.

8. Others pretend to have found out a method (incited thereto by an example *in Albert Gerards Invention Nouuelle en Algebre à Amſterdam* 1629.) ſo much, by comparing of Equations, to increaſe

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*Nota. *The Author* (*M.* Collins) *haveing explain'd* Mercators Logarithmotechnia *in Engliſh, and illuſtrated the elegant Doctrine thereof with Examples, hath likewiſe handled this work of Interpolation, and makes the* Logarithmes *true to* 25. *or* 30. *places of figures by meer Diviſion (or Proportion;) having herein advanc'd that Author's doctrine thereof by Diviſion, which (as 'tis there illuſtrated) did not* ſeem *to extend farr enough. This hath already been communicated, ſome Months ſince, to ſome of the Members of the* R. So-*ciety, and may be expected hereafter.*

creaſe or diminiſh the unknown Root of Equation, as to render it a whole number (or leſſe differing therefrom, than any Error aſſign'd,) and by *Albert Gerards* Method of *Aliquot parts* to find the ſame,and thereby the Root ſought, although it be a Mixt Number, Fraction, or Surd.

Probably this may ſympathiſe with what is promiſed by the Learned *Huddenius in Annexis Geometriæ Carteſianæ*, where he ſaith, he intended not *then* to publiſh certain Rules, he had ready; whereof one was, To find out all the irrational Roots both of Literal and Numeral Equations. This muſt be underſtood when ſuch Roots are *poſsible*; for 'tis certain, there are infinite Equations, whoſe Roots are no ways explicable, either in whole or mixt numbers, Fractions or Surds, and can be no otherwiſe explain'd, but by a *quàm proxime*.

9. The Author of this Narrative conſidering, that the *Conick Sections* may be projected from leſſer Circles placed on the Sphere, and thence eaſily (otherwiſe than hitherto hath been handled) deſcribed by Points, and that by their Interſections ſome Spherick Problem is determined, accordingly he found, that this following Problem according to the various Scituation of the Eye, and of the Projecting Plain, would take in all Caſes.

The Diſtances of an unknown Star are given from two Stars of known Declination and Right Aſcenſion; the Declination and Right Aſcenſion of the unknown Star is required.

And ſaith, he hath obſerved, that, admitting the Mechaniſme of dividing the Periphery of a Circle into any number of equal parts,or (which is equivalent) the Uſe of a Line of Chords, that this Problem, wherever the Eye be plac'd, may be reſolved by Plain Geometry, and yet the Ey ſhall be ſo plac'd, as to determin it by the Interſections of the Conick Sections; conſequently thoſe Points of Interſection (the Species and Poſition of the figures being given) may be found without deſcribing any more Points than thoſe ſought, and the Lengths of Ordinates falling from thence on the *Axes* of either figure calculated by mixt Trigonometry, and hence likewiſe the Roots of all *Cubick* and *Bi-quadratick* Equations found by *Trigonometry*.

For giving from the *Meſolabe* mention'd the *Scheme* that finds theſe Roots, it will then be required to fit thoſe *Sections* into *Cones*, which have their *Vertex* either in the Center; or an aſſigned point in the Surface of the Sphere, to which they

relate

relate as projected, and proceed to the reſolution of the Problem propoſ'd: And how to fit in thoſe Sections, ſee the 7. books of *Apollonius*, *Mydorgius*, the 3d. Volume of Des *Cartes's* Letters, *Leotaudi Geometrica practica*, *Anderſonii Exercitat. Geometricæ*.

As to the Problem it ſelf, it is determin'd on the Sphere by the Interſections of the two leſſer Circles of Diſtance, whoſe *Poles* are the known Starrs. And this Problem hath divers *Geometrick* ways of reſolution.

1. By *Plain Geometry* (in the ſenſe before-mentioned;) Suppoſing a Plain to touch the Sphere at the *North-pole*: if the Eye be at the *South-pole*, projecting thoſe Circles into the ſaid Plain, they are ſtill Circles (by reaſon of the ſub-contrary Sections of the Viſual Cones) whoſe Centers fall in the ſides of the Right-lin'd Angle, made by the Projected *Meridians*, that paſs through the known Starrs; and thus the Problem is eaſily ſolv'd in this manner.

2. If it be required to be performed by *Conick Geometry*; in one caſe it may be done, by placing the Ey at the Center of the Sphere, and projecting as before; to wit, when the longer *Axes* of the figures being produced concur above the *Vertex*; Here the *Problem* is determined by the Interſections of two Conick Sections (whereof a Circle cannot be one, unleſs its Center be in the *Axis* of the other figure.) And in this ſecond Caſe theſe points of Interſection fall in the ſame right line or projected *Meridian*, they did before, but at a more remote diſtance from the Pole-point, to wit, in the former Suppoſition, the Solar diſtance was meaſur'd by a Right line, that was the double Tangent of half the Arch; here it is the Tangent of the whole Arch. Hence it is evident, how one *Projection* may beget another, yea infinite others, altering the Scale; and how the leſſer Circles in the *Stereographick* Projection help to deſcribe the Conick Sections in the *Gnomonick* Projection: But (to reduce the matter to one common *radius*) if we ſuppoſe two Spheres equal, and ſo placed about the ſame *Axis*, that the Pole-point of the one ſhall paſs through the Center of the other, and the Touch-plain to paſs through the ſaid Center or Pole-point, and that a leſſer Circle hath the ſame poſition in the one as in the other; Then, if the Ey be at the South-Pole of the one, it is at the Center of the other; and any projected *Meridian* drawn from the projected Pole-point to paſs through both the projections of theſe leſ-

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ſer Circles, the diſtances of the Points of interſection are the Tangents of the half and the whole Arch of the Meridian ſo interſected. But as to the Points of Interſection, which determine the Problem propoſed, they may be found without the aid of the former way, from a *Gnoonmick* and *Stereographick* method of meaſuring and ſetting off the ſides and angles of Spherical Triangles in thoſe Projections, which is neceſſary in what follows.

3. If the Problem is to be perform'd by *Mixt Geometry*, as by a Circle and either a *Parabola*, *Hyperbola*, or *Ellipſis*, the Circle may be conceived to be the Sub-contrary Section of a Cone projected by the Eye at the *South-pole*, and any of the reſt of the Sections by the Eye at the Center of the Sphere.

4. If by any of the *Conick Sections* however poſited; the projecting Plain may remain the ſame, but the Eye muſt be in ſome other part of the *Surface* of the Sphere, and not in the *Axis*.

Theſe things were mention'd to invite the Learned to their Conſideration: I ſhall only further adde, that we cannot ſay, what may be expected from the labours and endeavours of divers Learned men of this Nation, particularly from Dr. *Wallis*, who hath ſo excellently reſolved and conſtructed all *Cubick Æquations* at the end of the firſt Treatiſe of his *Opera Mathematica* by aide of a Cubick *Parabolaſter*, mentioning, that by ſuch Curves the Roots of all Æquations may be found: And who hath promiſed a Treatiſe of *Algebra* and *Angular Sections*, wherein the Reader need not doubt to meet with ſatisfaction in theſe Myſteries. Nor ought we to omit the mentioning of the Modeſt and Learned Mr. *Barrow*, who (among many other excellent Subjects, and particularly his Opticks now, at the Preſs) hath perform'd, what the famous Italian Geometer *Mich. A. Ricci* hath promiſ'd in *Exercitat. Geometrica* (printed at *Rome* 1666. and lately reprinted here) about *Curves* of ſeveral degrees, that ſerve to determine and reſolve all *Æquations*: which hath likewiſe been done by other Learn'd men of this Nation.

An Account of Books.

I. PRÆLUDIA BOTANICA Roberti Moriſon *Scoti Aberdonenſis*. Londini, impenſis *Jac. Alleſtry*, 1669. in8o.

This Prelude of this Excellent Botaniſt hath two parts; The firſt gives us an Alphabetical Catalogue of all the Plants in the Royal